Pair Production of Heavy Quarkonium and $B_c(^*)$ Mesons at Hadron Colliders

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Abstract

We investigate the pair production of S-wave heavy quarkonium at the LHC in the color-singlet mechanism (CSM) and estimate the contribution from the gluon fragmentation process in the color-octet mechanism (COM) for comparison. With the matrix elements extracted previously in the leading order calculations, the numerical results show that the production rates are quite large for the pair production processes at the LHC. The p_t distribution of double J/ψ production in the CSM is dominant over that in the COM when p_t is smaller than about 8GeV. For the production of double Υ, the contribution of the COM is always larger than that in the CSM. The large differences in the theoretical predictions between the CSM and COM for the p_t distributions in the large p_t region are useful in clarifying the effects of COM on the quarkonium production. We also investigate the pair production of S-wave B_c and B_c^* mesons, and the measurement of these processes is useful to test the CSM and extract the LDMEs for the B_c and B_c^* mesons.

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I. INTRODUCTION

Heavy quarkonium provides a ideal system to investigate both the perturbative and non-perturbative aspects of quantum chromodynamics. Conventionally, the color-singlet mechanism (CSM) is used to describe the decay and production of heavy quarkonium[1]. In the CSM, the processes are factorized into two steps. Firstly the heavy quark pair are created perturbatively at short distances with the same color and angular momentum as the final quarkonium state, and then evolve into the quarkonium non-perturbatively at long-distances. There have been, however, some problems in the CSM, e.g., the infrared divergences in the calculation of the decay of P-wave quanrkonium[2] and the higher-order correction calculation of S-wave quanrkonium[3], and the surplus J/ψ production[4] of which the rate is much higher than that of the color-singlet prediction at the Tevatron. The non-relativistic quantum chromodynamics (NRQCD) factorization formalism[5], which was put forward by Bodwin, Braaten, and Lepage, overcame the infrared divergence difficulties in the color-singlet model[6], and gave the proper prediction for the charmonium production at the Tevatron[7]. In NRQCD, the heavy quark pair at short distances are not necessarily in the color-singlet state but can be in the states with different color and angular-momentum from that of the final state quarkonium. The color-octet pair can evolve to the color-singlet charmonium by emitting soft gluons. This is called the color-octet mechanism (COM).

Lots of work have been done to investigate the validity and limitation of the NRQCD formulism in heavy quarkonium production. The current experimental results on J/ψ photo-production at HERA are fairly well described by the NLO color singlet piece except the J/ψ polarizations [8, 9, 10]. The DELPHI data favor the NRQCD color-octet mechanism for J/ψ production $\gamma\gamma \to J/\psi X$ [11, 12]. The observed large cross sections of inclusive charmonium production at the Tevatron once gave strong support to the color-octet gluon fragmentation in NRQCD, but recently it is found that the NLO results in the color-singlet piece can bring an order of magnitude enhancement to the J/ψ production rate in the large p_t region[13] with longitudinally polarized J/ψ[14]. The theoretical prediction for p_t distribution of the Υ production can properly describe the Tevatron data by including the contributions from the NLO results and the real correction part at the next-to-next-to-leading-order in the CSM [15]. The cross sections of J/ψ exclusive and inclusive production in e^+e^- annihilation at B factories [16] are much larger than the LO NRQCD predictions[17, 18], but the discrepancies

seem to be resolved by considering the higher order effects: NLO QCD corrections[19, 20, 21] and relativistic corrections [22] without invoking the color-octet contributions[20] (discussions in the light-cone approach can be seen in [23]). Recent developments and related topics in quarkonium production can be found in Refs. [24, 25, 26].

The above mentioned developments in heavy quarkonium production indicate that the situation is far from being conclusive, and further tests for the color-singlet and color-octet mechanisms in NRQCD are still needed to clarify various problems involved in heavy quarkonium production.

In order to investigate the effects of the COM on the production of heavy quarkonium, it is useful to study processes which heavily depends on the production mechanism. The pair production of heavy quarkonium can serve as the desired process. In NRQCD, the gluon fragmentation gives the main contribution to the pair production of quarkonium in the large p_t region of the heavy quarkonium. In the pair production processes, there appear two long-distance matrix elements (LDMEs). So the difference of theoretical predictions between the CSM and COM could be more obvious. Moreover, because of charge-parity C conservation the gluon fusion processes $g+g \to J/\psi + \chi_c$ and $g+g \to J/\psi + \eta_c$ are forbidden in the CSM. But the gluon fragmentation in the COM can produce these associated final states. So to detect two final heavy quarkonium states with different C-parity may give a good way to test the COM.

The pair production of heavy quarkonium at hadron colliders has been studied by many authors. The color-octet gluon fragmentation into double charmonium at the Tevatron in NRQCD were considered as evidence for the COM [27]. The CSM prediction on the double charmonium production was made and it was found that the contribution with $p_t < 4GeV$ in the CSM is dominant[28]. Only in the large p_t region, the CSM and COM give manifestly different predictions for the double charmonium production. The Large Hadron Collider (LHC) is expected to produce a huge number of heavy quarkonium. Therefore, it is natural to investigate the pair production of heavy quarkonium at the LHC.

At the LHC, it is also interesting to study the production of the double heavy flavored mesons B_c and B_c^*. The $B_c(^*)$ meson production in hadron collisions has been studied in QCD[29, 30, 31]. Some study of $B_c(^*)$ pair production was also performed in pp and $\gamma\gamma$ collisions[32]. It is useful to extend the study to the pair production of the $B_c(^*)$ mesons at the LHC.

In this paper, we study the pair productions of heavy quarkonium at the Tevatron and LHC, including J/ψ, η_c, Υ and η_b in the CSM. For comparison, the color-octet contributions to the pair production of J/ψ and Υ are estimated by considering the gluon fragmentation process. We also investigate the pair productions of S-wave B_c and B_c^* mesons where there is no contribution from the gluon fragmentation process and the COM contributions are suppressed by the small v^2 (the relative velocity between quark and anti-quark). Therefore, these processes can give a better test of CSM and be used to extract the LDMEs of the B_c and B_c^* mesons.

The outline of our paper is as follows. In section 2, some of the definitions and formulas are given for deriving the cross sections of the processes. Then the numerical results are presented in section 3. Finally, in section 4 we give the summary.

II. THE FORMULATIONS

A. Color-singlet part

At hadron collider, the pair production of heavy quarkonium at the leading-order (LO) in the CSM have two subprocesses $g + g \to \mathcal{Q}_1 + \mathcal{Q}_2$ and $q + \bar{q} \to \mathcal{Q}_1 + \mathcal{Q}_2$. But we just consider the gluon fusion process in the calculation since it is the dominant one. There are 31 Feynman diagrams for the processes $g + g \to J/\psi + J/\psi$ and $g + g \to B_c(B_c^*) + \bar{B}_c(\bar{B}_c^*)$. The typical Feynman diagrams are presented in Fig. 1. For the process $g + g \to \eta_c + \eta_c$, there are additional 8 Feynman diagrams which are showed in Fig. 2. The Feynman diagrams for the process of Υ and η_b production are as same as the corresponding process of charmonium production.

Following the color-singlet factorization formalism, the amplitude of the pair production of S-wave heavy quarkonium is written as

$$
\begin{aligned}
&\mathcal{M}(a + b \to \mathcal{Q}_1 + \mathcal{Q}_2) \\
&= \sum_{s_1,s_2,s_3,s_4,i,j,k,l} N_1(\lambda|s_1, s_2) N_2(\lambda|s_3, s_4) \frac{\delta^{ij}}{\sqrt{N_c}} \frac{\delta^{kl}}{\sqrt{N_c}} \frac{R_1(0)}{\sqrt{4\pi}} \frac{R_2(0)}{\sqrt{4\pi}} \\
&\times \mathcal{M}(a + b \to Q_i \bar{Q}_j(\mathbf{p_1} = \mathbf{0}; s_1, s_2) + Q_k \bar{Q}_l(\mathbf{p_2} = \mathbf{0}; s_3, s_4))
\end{aligned} \tag{1}
$$

where the s_i is the spin of the heavy quark in the meson; $R_i(0)$ is the wave function at origin of the heavy quarkonium; $\frac{\delta^{ij}}{\sqrt{N_c}}$ is the color project operator; $N_i(\lambda|s_1, s_2)$ is the spin project
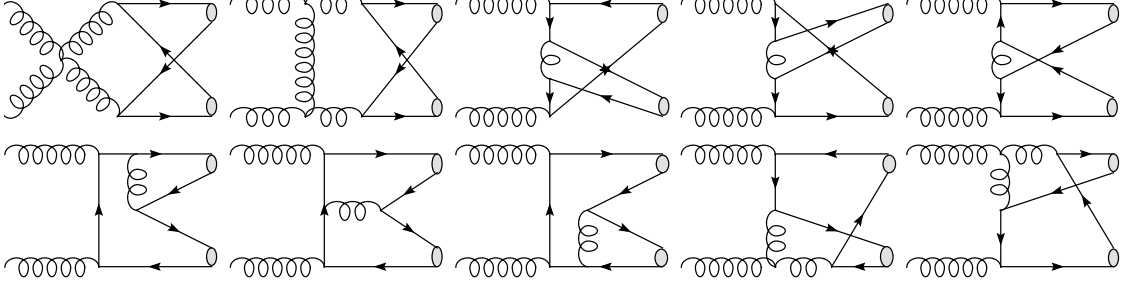
FIG. 1: The typical Feynman diagrams for $g + g \rightarrow J/\psi + J/\psi$. The others can be obtained by reversing the fermion lines or interchanging the initial gluons. As for $g + g \rightarrow B_c(B_c^*) + \bar{B}_c(\bar{B}_c^*)$, there are same diagrams.
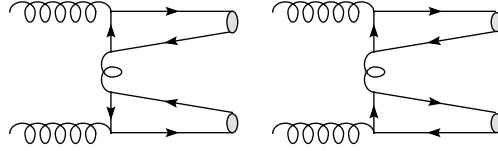

FIG. 2: The additional typical Feynman diagrams of $g + g \rightarrow \eta_c + \eta_c$. The others can be obtained by reversing the fermion lines or interchanging the initial gluons.

operator as following

$$N(\lambda|s_1, s_2) = \frac{\sqrt{M_{Q\bar{Q}}}\epsilon_\mu^\lambda \bar{v}(\mathbf{P}_Q, s_2)\gamma^\mu u(\mathbf{P}_{\bar{Q}}, s_1)}{4m_Q m_{\bar{Q}}}, \tag{2}$$

where $M_{Q\bar{Q}}$ is the mass of the heavy quarkonium.

We analytically calculate the amplitude square of these subprocesses and present the analytical formulas of the pair production of $\eta_c(\eta_b)$ and $J/\psi(\Upsilon)$ in appendix. For B_c and B_c^*, the analytical formulas are too tedious to be presented in this paper. The formula for $d\hat{\sigma}/dt$ on the double J/ψ production is the same as that in reference[28]. The final result can be obtained by convoluting the parton level cross section with the parton distribution function $f_{g/p}(x)$ as following

$$d\sigma(p + p(\bar{p}) \rightarrow \mathcal{Q}_1 \mathcal{Q}_2 + X) = \int dx_1 dx_2 f_{g_1/p}(x_1, \mu_f) f_{g_2/p(\bar{p})}(x_2, \mu_f) d\hat{\sigma}(g_1 + g_2 \rightarrow \mathcal{Q}_1 \mathcal{Q}_2, \mu_r) \tag{3}$$

where the μ_r and μ_f are the renormalization and factorization scale.

B. Color-octet Part

As a comparison, we naively estimate the pair production of J/ψ and Υ in the COM by using the similar way as in Ref.[27] in which the evolution of the fragmentation function was ignored. In the COM, a gluon can fragment into a $c\bar{c}$ pair with the quantum number ${}^3S_1^{(8)}$ and then hadronize into J/ψ. This process will give large contribution to the double heavy quarkonium production in the large p_t region and is expressed as

$$d\hat{\sigma}_{\mathcal{Q}_1+\mathcal{Q}_2} = \int_0^1 dz_1 \int_0^1 dz_2 D_{g\to\mathcal{Q}_1}(z_1, m_{Q_1}) D_{g\to\mathcal{Q}_2}(z_2, m_{Q_2}) d\hat{\sigma}_{gg}(E_1/z_1, E_2/z_2), \qquad (4)$$

where $\hat{\sigma}_{gg}$ is the cross section of two real gluon production; D is the fragmentation function for a gluon to fragment into a quarkonium. In NRQCD, this fragmentation function is written as[5]

$$D_{g\to\mathcal{Q}}(z, \mu^2) = \sum_n d_{g\to n}(z, \mu^2)\langle \mathcal{O}_n^H \rangle. \qquad (5)$$

The short distance coefficient can be calculated perturbatively and the result of the LO calculation is

$$d_{g\to\underline{8}^3S_1} = \frac{\pi\alpha_s(2m_Q)}{24m_Q^3}\delta(1-z). \qquad (6)$$

The contribution from $gg(q\bar{q}) \to gg$ subprocesses is calculated and the contributions from the feeddown of ψ', $\chi_{cJ}(1P)$, $\Upsilon(2S)$ and $\chi_{bJ}(1P)$ are also included. The final result is expressed as

$$
\begin{aligned}
d\hat{\sigma}_{\mathcal{Q}_1+\mathcal{Q}_2} = {} & d\hat{\sigma}_{gg}(\frac{\pi\alpha_s(4m_c^2)}{24m_c^3})^2[\langle O_8^{J/\psi}({}^3S_1)\rangle + \langle O_8^{\psi'}({}^3S_1)\rangle Br(\psi' \to J/\psi) \\
& + \sum_{J=0}^{2}(2J+1)\langle O_8^{\chi_{c0}(1P)}({}^3S_1)\rangle Br(\chi_{cJ} \to J/\psi)]^2.
\end{aligned}
\qquad (7)
$$

Here it is noteworthy that the identical particle factor "2" has been put in the calculation of $d\hat{\sigma}_{gg}$.

III. THE NUMERICAL RESULTS AND CONCLUSIONS

In calculating the numerical results, we choose the following parameters $M_c = 1.5\text{GeV}$ and $M_b = 4.9\text{GeV}$, and set the renormalization and factorization scale as $\mu_r = \mu_f = \sqrt{4m_Q^2 + p_t^2}$.

For the gluon fragmentation process, the renormalization and factorization scale are chosen as the transverse momentum $p_t(g)$ of the gluon with $p_t(J/\psi) \simeq p_t(g)$. The parton distribution of CTEQ6L1[33] is used. Therefore, the running of α_s is evaluated by the LO formula of CTEQ6. The center-of-mass energies of the Tevatron and LHC are 1.96TeV and 14TeV respectively. The pseudorapidity cuts on the final quarkonium states are chosen as $-0.6 < \eta < 0.6$ at the Tevatron and $-2.4 < \eta < 2.4$ at the LHC. We use the wave functions at origin that are calculated by using the logarithmic potential[34, 35] with quark masses almost the same as what we use. The values for them are listed as:

$$|R(0)|^2_{c\bar{c}(1S)} = 0.815 \text{GeV}^3,$$
$$|R(0)|^2_{b\bar{c}(1S)} = 1.508 \text{GeV}^3,$$
$$|R(0)|^2_{b\bar{b}(1S)} = 4.916 \text{GeV}^3. \tag{8}$$

In order to give the results of gluon fragmentation processes for comparison, the following color-octet matrix elements are used as the input parameters[36, 37]

$$\langle O_8^{J/\psi}(^3S_1)\rangle = 0.39 \times 10^{-2}\text{GeV}^3, \quad \langle O_8^{\Upsilon(1S)}(^3S_1)\rangle = 15 \times 10^{-2}\text{GeV}^3,$$
$$\langle O_8^{\psi'}(^3S_1)\rangle = 0.37 \times 10^{-2}\text{GeV}^3, \quad \langle O_8^{\Upsilon(2S)}(^3S_1)\rangle = 4.5 \times 10^{-2}\text{GeV}^3,$$
$$\langle O_8^{\chi_{c0}(1P)}(^3S_1)\rangle = 0.19 \times 10^{-2}\text{GeV}^3, \quad \langle O_8^{\chi_{b0}(1P)}(^3S_1)\rangle = 4.0 \times 10^{-2}\text{GeV}^3. \tag{9}$$

These LDMEs are extracted from the matching between the LO NRQCD predictions and the Tevatron data by using the CTEQ5L parton distribution function with LO α_s running. From the LO formula of α_s running of CTEQ6, the corresponding α_s in the fragmentation function for the J/ψ and Υ are chosen as $\alpha_s(M_{J/\psi})$=0.286, $\alpha_s(M_\Upsilon)$=0.201 respectively. The branching ratios in Eq. (7) are taken from the PDG08[38].

In the Table I, we give the cross sections of pair production of J/ψ (Υ) and B_c (B_c^*) at the Tevatron and LHC with $p_t > 3$GeV in the CSM. From the table, the cross section of each process is enhanced by an order or more in magnitude at the LHC than that at the Tevatron. Therefore, the LHC will be a good place to study the pair production processes carefully.

Fig. 3 shows the p_t distribution of the pair production for J/ψ and η_c at the Tevatron and LHC. The result of the gluon fragmentation process in the COM is also plotted in the figure. We can see that whether at the Tevatron or at the LHC the p_t distributions of J/ψ and η_c are similar and the numerical results at the LHC are enhanced by an order or more

TABLE I: The cross sections of pair production of J/ψ, Υ, B_c and B_c^* at the Tevatron and LHC with $p_t > 3$GeV.

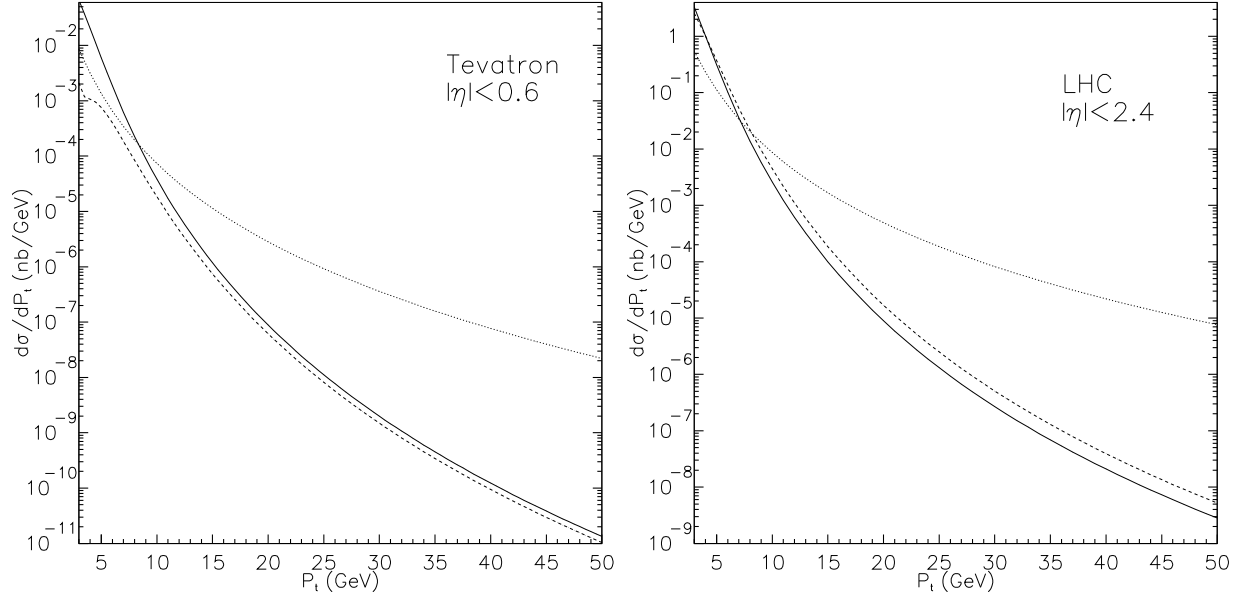| Final States	$\sigma_{Tevatron}[nb]$	$\sigma_{LHC}[nb]$
$\eta_c\eta_c$	3.32×10^{-3}	2.73
$J/\psi J/\psi$	5.63×10^{-2}	2.83
$\eta_b\eta_b$	1.87×10^{-5}	7.36×10^{-3}
$\Upsilon\Upsilon$	1.23×10^{-4}	1.51×10^{-2}
$B_c\bar{B}_c$	3.86×10^{-3}	2.72×10^{-1}
$B_c\bar{B}_c^*$	1.00×10^{-3}	8.37×10^{-2}
$B_c^*\bar{B}_c^*$	8.23×10^{-3}	7.08×10^{-1}

FIG. 3: The pair production of J/ψ (solid line) and η_c (dashed line) in the CSM at the hadron colliders. The dotted line corresponds to the pair production of J/ψ that come from the gluon fragmentation process in the COM.
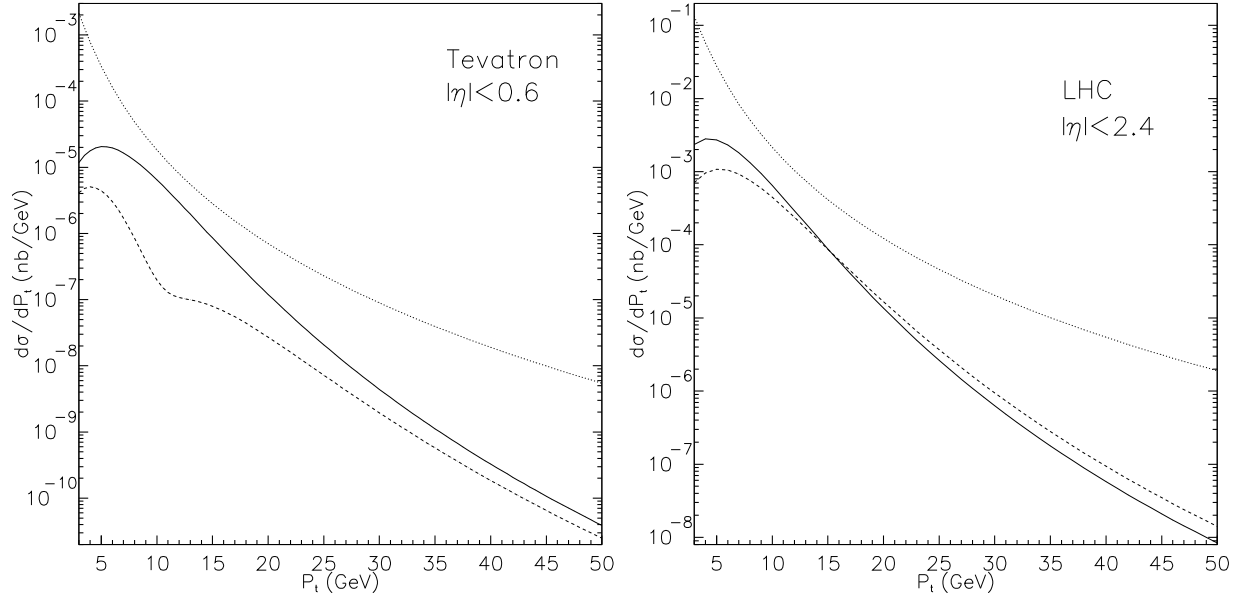
FIG. 4: The pair production of Υ (solid line) and η_b (dashed line) in the CSM at the hadron colliders. The dotted line corresponds to the pair production of Υ that come from the gluon fragmentation process in the COM.

in magnitude at large p_t region than that at the Tevatron. Therefor, the LHC will provide a chance to measure the p_t distribution of J/ψ pair production. Comparing the J/ψ pair production in the CSM with that in the COM, the formal is dominant as the p_t is smaller than about 8GeV. And the result in the COM become dominant and even larger than that in the CSM for three orders in the large p_t region.

The p_t distributions of Υ and η_b production are shown in Fig. 4. The p_t distribution is enhanced by an order or more in magnitude at the LHC than that at the Tevatron. But unlike the case of J/ψ pair production, the pair production of Υ in the COM dominate over that in the CSM in the whole p_t region, and even is two order or more in magnitude larger than that in the CSM at large p_t.

Because the B_c and B_c^* are consist of quarks with different flavors, there is no contribution from gluon fragmentation processes in the COM. So in Fig. 5, we only give the p_t distribution of the pair production for $B_c \bar{B}_c$, $B_c \bar{B}_c^*$ and $B_c^* \bar{B}_c^*$ in the CSM. The pair production of $B_c^* \bar{B}_c^*$ is dominant in the whole p_t region. There are little difference between the production of the three final states with double-heavy flavor mesons in the CSM.

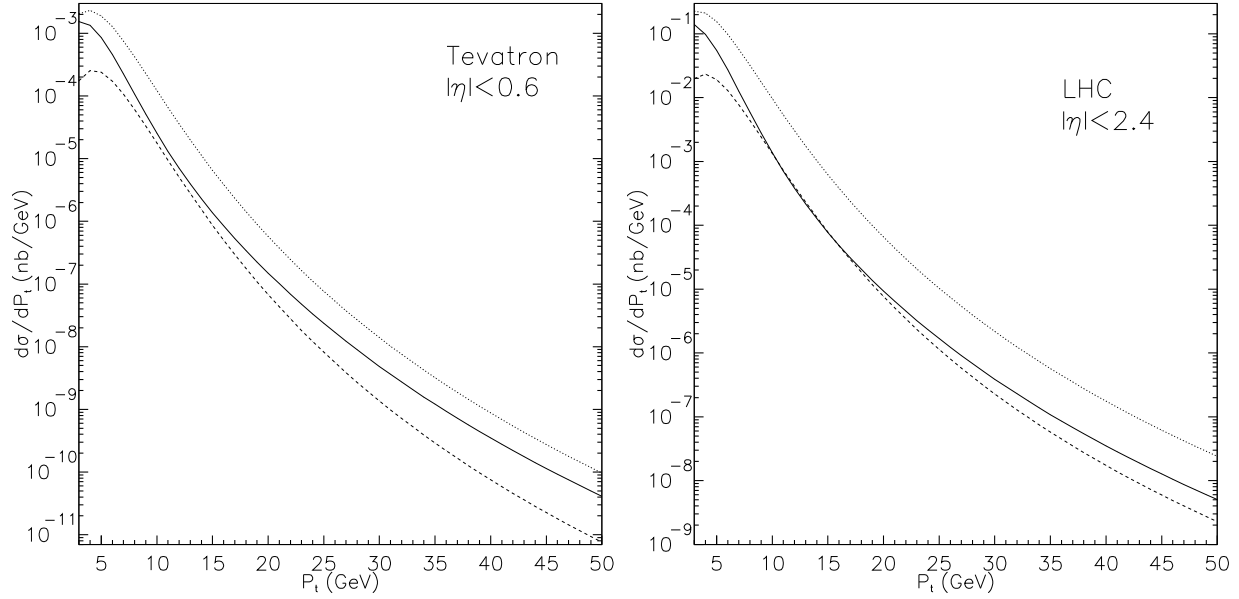From the Table I and the figures, it can be seen that all of the cross sections are enhanced

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FIG. 5: The production of $B_c\bar{B}_c$ (solid line), $B_c\bar{B}_c^*$ (dashed line) and $B_c^*\bar{B}_c^*$ (dotted line) in the CSM at the Tevatron and LHC.

when the center-of-mass energy is increased. This is because that with the fixed p_t, the larger the $\sqrt{s}$ is, the smaller the momentum fraction x of the parton is. In small x region, the parton distribution function of gluon increases rapidly.

IV. SUMMARY

In this paper, we have investigated the leading order pair production of S-wave heavy quarkonium at hadron colliders in the color-singlet mechanism (CSM) and estimated the contributions from the gluon fragmentation process in the color-octet mechanism (COM) for comparison. With the matrix elements extracted previously in leading order calculations, the numerical results show that the production rates are quite large for the pair production processes at the LHC. The p_t distribution of double J/ψ production in the CSM is dominant over that in the COM when p_t is smaller than about 8 GeV. For the production of double Υ, the contribution of the COM is always larger than that in the CSM. There are large differences in the theoretical predictions between the CSM and COM for the p_t distributions in the large p_t region, and this is useful in clarifying the effects of COM on the quarkonium production. Furthermore, since to produce a pair of quarkoniums with different C-parity is forbidden in the CSM at the leading-order, the observation of these processes could be a

positive support for the COM. We also investigate the pair productions of S-wave B_c and B_c^* mesons, and the measurement of these processes may be useful to test the CSM and extract the LDMEs for the B_c and B_c^* mesons.

After our work was completed [39], a paper appeared [40], in which Qiao, Sun, and Sun calculated the double J/ψ production at the LHC. They focused on the polarizations of the double J/ψ. We focused on the cross sections of double heavy quarkonia J/ψ, η_c, Υ, η_b, as well as the double heavy flavored B_c, and B_c^* mesons. Both the two papers discuss the test of the COM. Our color octet and color singlet double J/ψ cross sections are consistent with their result [39].

Acknowledgments

Appendix

In this appendix we give the parton level cross section for the pair production of $\eta_c(\eta_b)$ and $J/\psi(\Upsilon)$ respectively. Here m is the mass of the corresponding meson and the s, t, u are the Mandelstam variables defined as

$$
\begin{aligned}
s &= (k_1 + k_2)^2, \\
t &= (k_1 - P)^2, \\
u &= (k_2 - P)^2,
\end{aligned}
\tag{10}
$$

where k_1, k_2 and P are the momenta of the initial gluons and one of the final quarkonium states.

1. $g + g \rightarrow \eta_c + \eta_c$

$$\frac{d\hat{\sigma}}{dt} = \frac{\alpha_s{}^4\pi|R(0)|^4}{162m^2 s^8 (m^2-t)^4 t^2 (-2m^2+s+t)^2 (-m^2+s+t)^4}(331776t^2 m^{28} - 239616t^2(7s$$

$$+18t)m^{26} + 256t(27s^3 + 14257ts^2 + 81792t^2s + 101412t^3)m^{24} - 64t(543s^4 + 72538ts^3$$

$$+687780t^2s^2 + 1876968t^3s + 1498176t^4)m^{22} + 4(9s^6 + 18144ts^5 + 18144ts^5$$

$$+1018456t^2s^4 + 13409824t^3s^3 + 60363088t^4s^2 + 104825088t^5s + 60673536t^6)m^{20}$$

$$-4(45s^7 + 20364ts^6 + 693676t^2s^5 + 10769920t^3s^4 + 70075680t^4s^3 + 199182848t^5s^2$$

$$+248330880t^6s + 111310848t^7)m^{18} + (369s^8 + 53568ts^7 + 1532448t^2s^6 + 24574336t^3s^5$$

$$+210621248t^4s^4 + 870605312t^5s^3 + 1761630976t^6s^2 + 1686196224t^7s + 610384896t^8)m^{16}$$

$$-4(99s^9 + 5496ts^8 + 172056t^2s^7 + 2626424t^3s^6 + 27129120t^4s^5 + 152150816t^5s^4$$

$$+446139360t^6s^3 + 687430400t^7s^2 + 527406336t^8s + 158754816t^9)m^{14} + 2(117s^{10}$$

$$+3420ts^9 + 131584t^2s^8 + 1843176t^3s^7 + 20064012t^4s^6 + 143086496t^5s^5 + 571404544t^6s^4$$

$$+1267269888t^7s^3 + 1553349056t^8s^2 + 984517632t^9s + 251817984t^{10})m^{12} - 4(18s^{11}$$

$$+588ts^{10} + 21495t^2s^9 + 304486t^3s^8 + 2927138t^4s^7 + 23491008t^5s^6 + 120913176t^6s^5$$

$$+363365760t^7s^4 + 636265120t^8s^3 + 639890816t^9s^2 + 342648576t^{10}s + 75727872t^{11})m^{10}$$

$$+(9s^{12} + 810ts^{11} + 20676t^2s^{10} + 360708t^3s^9 + 3111538t^4s^8 + 23098608t^5s^7$$

$$+138861808t^6s^6 + 537887488t^7s^5 + 1266810688t^8s^4 + 1806816000t^9s^3 + 1525860352t^{10}s^2$$

$$+702443520t^{11}s + 135945216t^{12})m^8 - 4t(45s^{12} + 903ts^{11} + 18636t^2s^{10} + 181253t^3s^9$$

$$+1173996t^4s^8 + 7001216t^5s^7 + 32407688t^6s^6 + 98050752t^7s^5 + 186725360t^8s^4$$

$$+222162048t^9s^3 + 160423104t^{10}s^2 + 64384128t^{11}s + 11031552t^{12})m^6 + 2t(9s^{13} + 396ts^{12}$$

$$+4962t^2s^{11} + 58165t^3s^{10} + 384536t^4s^9 + 2056988t^5s^8 + 10083672t^6s^7 + 37041746t^7s^6$$

$$+90302176t^8s^5 + 142532816t^9s^4 + 144160704t^{10}s^3 + 90317984t^{11}s^2 + 31965696t^{12}s$$

$$+4893696t^{13})m^4 - 4t^2(45s^{13} + 312ts^{12} + 2799t^2s^{11} + 20392t^3s^{10} + 103737t^4s^9$$

$$+480982t^5s^8 + 1974841t^6s^7 + 5914228t^7s^6 + 11914204t^8s^5 + 15898752t^9s^4$$

$$+13880896t^{10}s^3 + 7636608t^{11}s^2 + 2406528t^{12}s + 331776t^{13})m^2 + t^2(s+t)^2(18s^{12}$$

$$+90ts^{11} + 447t^2s^{10} + 3162t^3s^9 + 14485t^4s^8 + 57520t^5s^7 + 241296t^6s^6 + 755200t^7s^5$$

$$+1481344t^8s^4 + 1774080t^9s^3 + 1267200t^{10}s^2 + 497664t^{11}s + 82944t^{12})) \qquad (11)$$

2. $g + g \rightarrow J/\psi + J/\psi$

$$\frac{d\hat{\sigma}}{dt} = \frac{16\alpha_s{}^4\pi|R(0)|^4}{81m^2s^8(m^2-t)^4(-m^2+s+t)^4}(7776m^{24} - 432(73s + 216t)m^{22}$$

$$+6(9085s^2 + 60336ts + 85536t^2)m^{20} - 16(3629s^3 + 37686ts^2 + 117855t^2s$$

$$+106920t^3)m^{18} + 4(11927s^4 + 151588ts^3 + 745674t^2s^2 + 1470960t^3s$$

$$+962280t^4)m^{16} - 4(7761s^5 + 109608ts^4 + 699467t^2s^3 + 2173908t^3s^2$$

$$+3055320t^4s + 1539648t^5)m^{14} + 2(6952s^6 + 117893ts^5 + 897043t^2s^4$$

$$+3741980t^3s^3 + 8278410t^4s^2 + 8872416t^5s + 3592512t^6)m^{12} - 2(1899s^7$$

$$+43398ts^6 + 405618t^2s^5 + 2113568t^3s^4 + 6394090t^4s^3 + 10762584t^5s^2$$

$$+9189936t^6s + 3079296t^7)m^{10} + (587s^8 + 19710ts^7 + 244772t^2s^6$$

$$+1603468t^3s^5 + 6229962t^4s^4 + 14478304t^5s^3 + 19359816t^6s^2 + 13582080t^7s$$

$$+3849120t^8)m^8 - 2(20s^9 + 1185ts^8 + 22153t^2s^7 + 193780t^3s^6 + 965358t^4s^5$$

$$+2928368t^5s^4 + 5431786t^6s^3 + 5949528t^7s^2 + 3508920t^8s + 855360t^9)m^6$$

$$+(s^{10} + 76ts^9 + 3756t^2s^8 + 52062t^3s^7 + 353472t^4s^6 + 1398834t^5s^5$$

$$+3421754t^6s^4 + 5210968t^7s^3 + 4784622t^8s^2 + 2414880t^9s + 513216t^{10})m^4$$

$$-4t^2(s+t)^2(9s^7 + 649ts^6 + 6460t^2s^5 + 29630t^3s^4 + 74435t^4s^3 + 105156t^5s^2$$

$$+77868t^6s + 23328t^7)m^2 + 2t^4(s+t)^4(349s^4 + 2304ts^3 + 6192t^2s^2$$

$$+7776t^3s + 3888t^4)) \tag{12}$$

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